LAB 10 – Conservation of Energy and Momentum

Group: __________

Names: _________________________ _________________________ _________________________

(Principle Coordinator) (Lab Partner) (Lab Partner)

Goals:
- Understand and apply the concepts of momentum.
- Re-enforce the concepts of kinetic energy and gravitational potential energy.
- Learn how to apply, observe and test the laws of conservation of energy and momentum.

Scenario and Strategy:
At a shooting range, you witness a probe of marksmanship when a bullet hits a piece of wood suspended at the end of a long thread. The bullet gets embedded into the wooden block and then the target swings up as a pendulum at a maximum angle that you record carefully. Suddenly you have an epiphany and realize that physics will allow you to calculate the speed of the bullet at the moment when it hits the target based on readily measurable characteristics of the pendulum and its swinging angle. So, you decide to test the theory in our lab.

- The theoretical model is based on conservation of momentum and energy, whereas the experimental arrangement will include a ballistic pendulum
- You will apply conservation of momentum to the perfectly inelastic collision between a ball and a pendulum
- Then you will apply conservation of mechanical energy to the upward swing of the ball-pendulum
- Combining the equations from applying the conservation laws will result in an expression for the initial speed of the ball before collision in terms of experimentally measurable quantities: this will lead to an experimental speed
- This experimental speed is to be compared to the muzzle speed of the launcher measured in LAB05 (Projectile Motion)

Equipment and Handling:
- Ballistic pendulum with ball and ramrod

**Ballistic Pendulum**
- You have already used this apparatus to launch projectiles
- However, to observe conservation of momentum, in this lab you will also use its pendulum part
- Push the ball against the piston to load the launcher to the second speed setting
- Once the ball is shot, it is immediately captured and trapped by a catcher pendulum
- Then, the catcher with the ball inside will swing upwards to a height \( h \), between points A and B (you will use these notations further below)
- On its way upward, the pendulum arm will drive an indicator along a protractor, such that the maximum angle displacement \( \theta \) of the pendulum can be recorded
- To calculate the maximum height \( h \) reached by the pendulum relative to the position of the collision, you will use the formula devised in the pre-lab
- In this formula you will use the length \( L \) of the pendulum arm measured to its center of mass when the ball is the catcher
- Also measure the mass of the ball \( m_b \) and the mass \( m_p \) of the pendulum (alone) using the electronic balance. Measure each mass twice and then average.
PART 1: Theoretical Framework

A. Conservation of Momentum in the Collision

- When the ball collides with the pendulum, its motion is redistributed to the combined object.
- In the table below you have the notations for the quantities involved in the inelastic collision.
- If the quantity is directly measurable (such as the masses and the length pendulum arm), proceed with the measurement and fill the values in.
- If the quantity is not known, just enter NA (Not Available) in the value cell. These are your unknowns in this phase. After you find these quantities, do not return to fill them in. These are just to emphasize known and unknowns in the theoretical calculation.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the ball, ( m_b )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass of the pendulum, ( m_p )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Velocity of ball immediately before collision, ( v_b )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Velocity of pendulum before collision, ( v_p )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Velocity of ball and pendulum immediately after collision, ( v_A )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Based on conservation of momentum, use the space below to find an expression relating the quantities in the table:
  - Do not use numbers in this space. Only symbols.
  - Denote \( p_b \) and \( p_p \) the momenta of the ball and pendulum before the collision, and \( p_{bA} \) and \( p_{pA} \) their momenta after collision. Then use these symbols to write the conservation of net momentum in the collision.
  - Replace the momenta with their definitions in terms of mass and velocity:
    \[ p = m v \]
  - Rearrange the equality to write \( v_b \) in terms of masses and \( v_A \). This is your first equation.

- Note that in order to find \( v_b \) you need the velocity \( v_A \) of the composite pendulum immediately after collision. For that you will need to use the conservation of energy in the ensuing swing where \( v_A \) is initial velocity.
- Go to the next page.
B. Conservation of Mechanical Energy in the Swing

- As the composite pendulum swings up, its mechanical energy is conserved while its nature changes from purely kinetic energy immediately after collision to purely gravitational at the maximum height.
- In the table below you have the notations for the quantities involved in the swing, besides the quantities defined in the previous table.
- If the quantity is not known, just enter NA (Not Available) in the value cell. These are your unknowns in this phase.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial height, $y_A$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum height, $y_B$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pendulum arm length, $L$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum displacement angle, $\theta$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Velocity of ball and pendulum at maximum height, $v_B$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Based on conservation of energy, use the space below to find an expression for $v_A$ in terms of measurable quantities:
  - Do not use numbers in this space. Only symbols.
  - Denote $E_A$ the mechanical energy of the ball and pendulum immediately after the collision, and $E_B$ their energy at maximum height. Then use these symbols to write the conservation of energy between the bottom of the swing (point A) and the top of the swing (point B)
  - Replace the mechanical energies with their definitions in terms of the respective kinetic energies ($K$) and gravitational energies ($U_g$):
    $$ E = K + U_g = \frac{1}{2} mv^2 + mgy $$
    (Adapt the notation. For instance, the energies in point B are $K_B$ and $U_B$).
  - Rearrange the equality to write $v_A$ in terms of masses and $y$.
  - Rewrite the equation by replacing $y_B$ in terms of $L$ and $\theta$. This is your second equation.

C. Combine the Equations

- Substitute $v_A$ from the second equation into the first equation to find $v_b$ in terms of measurable quantities.

- Now proceed with the experiment.
PART 2: Experiment and Discussions

- Fire the ball five times and record the maximum displacement angle. Fill the data in the table below:

<table>
<thead>
<tr>
<th>Trial #</th>
<th>Angle (°)</th>
<th>Height (m)</th>
<th>Mass of ball (kg)</th>
<th>Mass of pendulum (kg)</th>
<th>Mass of ball + pendulum (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Use the space below to show the numerical calculation for $v_b$, the average of velocities, and the percent difference.

$v_b$ calculation: | Calculation of average muzzle speed: | Percent difference: | Add the masses with the appropriate number of significant figures

- In the table below fill the speed of the ball before the collision, the muzzle speed of the ball measured in LAB05 (Projectile Motion), their average, and the percent difference between them.

<table>
<thead>
<tr>
<th>Velocity</th>
<th>Value (m/s)</th>
<th>Average speed (m/s)</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of ball before collision, $v_b$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Muzzle speed from LAB05, $v$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Discussion and Conclusions

Indicate several sources of errors in this experiment. Refer to the assumptions made when using conservation of energy. Which velocity value is more trustworthy when comparing to the projectile-motion experiment? Summarize what you learned in this experiment.

You lab report should include this form completed with data, calculations and discussions.