LAB 09 – Elastic Forces and Energy

Group: __________

Names: _____________________________________________

(Principal Coordinator) (Lab Partner) (Lab Partner)

Goals:
- Understand and apply Hooke’s Law of elasticity.
- Understand and apply the concept of elastic potential energy.
- Review the concept of mechanical energy and apply, observe and test the law of conservation of energy.

Scenario and Strategy:
You observe a child playing with a toy gun firing plastic balls. To load the gun, the child pushes the ball against a coil spring inside the gun barrel. For transcendental philosophical reasons, you become curious about the muzzle speed of the projectiles. You wonder, how could one estimate it based exclusively on easy to measure quantities? For instance, one can readily measure the mass of the ball and the compression of the spring. You need a trustable model involving the elasticity of the spring and the mechanism of motion transfer from the spring to the ball.
- In PART 1, you will generate a model for calculating the muzzle speed based on conservation of energy. All the necessary quantities will be readily measurable, except the elastic constant $k$ of the spring in the gun
- In PART 2 you will assume the spring to be ideal such that Hooke’s Law is applicable for the full range of compressions. Then you will use this law to estimate the spring constant $k$
- In PART 3, you will use the results from PART 1 and 2 to predict the muzzle speed of the projectile. Then, to confirm the model, you will use the same spring to fire a ball and measure the muzzle speed directly and compare it to the speed computed theoretically

Equipment and Handling:
- PASCO Ballistic Pendulum with ball and ramrod
- Smart Timer with Double Photogates accessory
- Caliper and holder
- Cylinder plug, weights, clamps

PART 2
- In this part, the launcher will be set in vertical position, with the caliper mechanism attached to it as in the adjacent figure
- When handling the caliper, make sure that you do not slide it up or down in its clamp

PART 3
- In this part, the launcher will be set in horizontal position, with the caliper mechanism removed and replaced with the double-gate system
- A ball will be launched and its muzzle speed will be measured using the two photogates separated by distance $d$. The gates are connected to the Smart Timer on the [Time:Two-Gates] setting; gate nearest to the muzzle to Port 1, the other to Port 2
- The gates will measure the time interval $t$ between the first and second beam interruptions
- Therefore, assuming a constant velocity in the respective interval, it can be estimated as an average speed using $v = \frac{d}{t}$
PART 1: Generate a model for finding the muzzle speed of the ball

- To calculate the muzzle speed of the projectile, you have to develop a formula based on the conservation of energy of the spring during the launch.
- The configurations of interest are: A) when the ball is pushed against the spring, still at rest while the spring is at maximum compression; B) the ball is at the exit, moving with muzzle speed, and the spring is relaxed.
- In the table below you have the notations for the quantities involved in the process, and one given quantity (the mass of the internal spring mechanism, \( m_s \)).
- If the quantity is directly measurable (such as the ball mass), proceed with the measurement and fill the value in. Use your ingenuity to find the spring compression relative to the initial position of the blue ring inside the launcher. For instance, use the ramrod to measure the initial and final positions of the blue ring and subtract them.
- If the quantity is not known, just enter NA (Not Available) in the value cell. This is the unknown to derive using the model.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the internal mechanism, ( m_s )</td>
<td>0.063 kg</td>
<td></td>
</tr>
<tr>
<td>Mass of the ball, ( m_b )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net mass, ( m = m_s + m_b )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastic constant from PART 2, ( k )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial spring compression (use the second spring setting), ( y_A )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final spring compression, ( y_B )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial speed of the ball, ( v_A )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final speed of the ball (muzzle speed), ( v_B )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Assuming that the mechanical energy is conserved, use the space below to find an expression relating the quantities in the table:

- Do not use numbers in this space. Only symbols.
- Denote \( E_A \) and \( E_B \) the mechanical energies of the spring-ball system in configurations A and B.
- Set them equal and then write them in terms of the kinetic and elastic potential energies, similar to Equation 7 in the Pre-Lab. Based on the values in the table, eliminate the terms which are zero.
- Rearrange the equality to write \( v_B \) in terms of known quantities in the table. This is your model for the muzzle speed.

\[
E_A = k, m_s, v_A = 0
\]

\[
E_B = v_B = y_A, y_B = 0
\]

Write the final formula for the muzzle speed here, in terms of \( y_A, k \) and the net mass of the system \( m = m_s + m_b \). Note that everything can be easily measured, except constant \( k \). Go to PART 2 to figure it out.

\[
v_B = \quad
\]
PART 2: Estimate the spring constant $k$

1. The strategy
   - According to Hooke’s Law, the compression of the spring is proportional to the elastic force in the spring, so if the force is graphed versus the compression, the spring constant is the slope of the graph
   - You will estimate the elastic force by placing weights in equilibrium on the spring, such that the elastic force matches the weight
   - The compressions will be measured using the caliper

2. Data collection
   - Open an Excel spreadsheet and reproduce the table below. (At the end of the lab transfer all the numbers from the Excel spreadsheet here, in the table on the Lab-Form)
   - Place the plug inside the launcher barrel. It is relatively light, so it won’t depress the spring significantly
   - Make sure that the lower edge of the tape on the plug is at about the same level as the caliper upper jaw (if it is not, ask your instructor to fix it). The edge will be the reference position for compression measurements
   - Place the weight hanger on top of the plug; its mass is about 50 g. One by one, add 10 weight rings on the hanger, each adding a 100-gram mass to the overall mass. While making sure that the caliper stays vertical, slide the caliper lower jaw down to the tape edge and read the compression for each mass. Enter the data in the table

<table>
<thead>
<tr>
<th>#</th>
<th>Mass $m$ (kg)</th>
<th>Elastic Force $F = mg$ (N)</th>
<th>Compression $y$ (m)</th>
<th>Elastic Constant $k$ (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Plot the data, process, and discuss
   - Plot the Elastic Force versus the Compression in Excel. Ornate the chart: add a title and label the axes
   - Generate a linear best fit. Set Excel to show the equation for the best fit
   - From this equation, you can extract the slope of the linear distribution. This is your experimental Elastic Constant
   - In the space below, discuss this part of the experiment, such as the reason for using the slope of the $F$-$y$ graph rather that only one pair of Force-Compression values to estimate $k$. Emphasize some errors that may have affected your measurement. For instance, how valid is the assumption that the spring is ideal?
PART 3: Calculate the theoretical speed and verify it experimentally

- Plug the values from the table into the formula in PART 1 to calculate the theoretical muzzle speed $v_B$ numerically. Fill it in the table below. Yet, to see how much can you trust this model, you will have to check it out experimentally.

1. Setup for model validation
   - Note that the clamp holding the caliper is kept fastened to the shaft by a screw. Untighten it and carefully slide the clamp off the shaft.
   - Rotate the launcher in horizontal position. The shaft is now vertically underneath the barrel.
   - Slide the square nut of the double-gate bracket along the rail underneath the barrel. Fasten the screw.
   - Make sure that the gates are parallel with each other and their beam line aligns with the diameter of the ball. Measure the distance $d$ between the beams and fill the value in the table below.
   - Set the smart timer as detailed on the cover page.

2. Experimental model validation
   - Fire the projectile three time using the second speed setting in the launcher.
   - Record the times in the table below and use their average to calculate an experimental muzzle speed.
   - Calculate the percent error $%E$ to compare this experimental value with the theoretical value computed above.

<table>
<thead>
<tr>
<th>Beam separation $d$ (m)</th>
<th>Trials for time $t$ between gates (s)</th>
<th>Average time $t$ (s)</th>
<th>Experimental speed $v_{exp}$ (m/s)</th>
<th>Theoretical speed $v_B$ (m/s)</th>
<th>Percent Error $%E$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Calculations
   - In the space below, show your calculation for the experimental muzzle speed and the percent error:

   $$v_{exp} = \quad \%E =$$

4. Discuss the validity of the model
   - In the space below, list thoroughly the errors that may have affected your measurement. Based on the value and associated percent error, was the model confirmed or not?

CONCLUSIONS
   - Discuss the subject, the procedure and the results of your experiment. Summarize what you learned from this lab unit.

You lab report should include this form – completed with data, calculations and discussions – and a printout with the Excel spreadsheet used to plot the data in PART 1.